Discrete Mathematics

1: Basic Logic: Propositions

(c) Marcin Sydow

Contents

Proposition

Truth: 1 (T)

False: 0 (F)

Proposition:

*a declarative statement that is **true** or that is **false***

Proposition is a building block of logic.

The area of logic that deals with propositions is called **propositional calculus**.

Examples

Is the following sentence a proposition?

- “2+2=4”

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Is every statement a proposition?

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Not every statement is a proposition!

Truth value of a proposition

We use letters to denote propositional variables: p, q, r, etc.

We use constants:

- **1** (or **T**) for “true”
- **0** (or **F**) for “false”

The **truth value** $v(p)$ of a proposition p is true if the proposition is true, or false if p is false.

Example:
if p is the following proposition “$0 < 1$”, then $v(p) = 1$ (true)

Logical operators

Logical operators are used to create **compound** propositions by combining other propositions.

Assume, that p, q are variables representing some propositions:

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- **implication**: $p \rightarrow q$ (“if p then q”) (conditional statement)

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- **exclusive or**: $p \oplus q$ ("either p or q") ("xor")
- **implication**: $p \rightarrow q$ ("if p then q") (conditional statement)
- **biconditional**: $p \leftrightarrow q$ ("p if and only if q") (equivalence)

The operators can be defined with "truth tables" - tables specifying what is the true value of the given operator depending on the truth values of its operands

Negation

$\neg p$: "not p", "it is not the case that p", etc.

The negation of a proposition is true only if the proposition is false and otherwise.

Truth table defining the negation depending on the value of its operand p:

p:	$\neg p$:
T	F
F	T

Examples:
p: "Warsaw is a capital of Poland", $\neg p$: "Warsaw is not a capital of Poland"
p: "$2 < 3$", $\neg p$: "$2 \geq 3$", etc.

Disjuntion

$p \vee q$: "p OR q"
The disjunction is true if at least one of its operands is true:

p:	q:	$p \vee q$:
F	F	F
F	T	T
T	F	T
T	T	T

Example:
"Snow is warm or Warsaw is capital of Poland"

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Example:

"Snow is warm or Warsaw is capital of Poland"(true)

p: "$3 > 3$" (false)

q: "$3 = 3$" (true)

$p \vee q$: ?

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p:	q:	$p \vee q$:
F	F	F
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Example:
“Snow is warm or Warsaw is capital of Poland”(true)

p: “$3 > 3$” (false)
q: “$3 = 3$” (true)
$p \vee q$: ? (true: “$3 \geq 3$”)

Conjunction

$p \wedge q$: "p AND q"

The conjunction is true only if both its operands are true

p:	q:	$p \wedge q$:
F	F	F
F	T	F
T	F	F
T	T	T

Examples:
p: "Kraków is in Poland" (true)
q: "Rome is in Poland" (false)
$p \wedge q$: "Kraków is in Poland and Rome is in Poland"?

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q: "Rome is in Poland" (false)
$p \wedge q$: "Kraków is in Poland and Rome is in Poland"?(false)

Exclusive or

$p \oplus q$: "p xor q"
Exclusive or is true only if exactly 1 of its operands is true.

p:	q	$p \oplus q$:
F	F	F
F	T	T
T	F	T
T	T	F

Example:
"$(2 > 1) \oplus (2 > 0)$" ?

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p:	q	$p \oplus q$:
F	F	F
F	T	T
T	F	T
T	T	F

Example:
"$(2 > 1) \oplus (2 > 0)$" ?(false)

Implication (conditional statement)

"$p \rightarrow q$": "p implies q", "if p then q", etc.

The first proposition in the implication is called "hypothesis" (or "antecedent" or "premise")

The second proposition in the implication is called "conclusion" (or "consequence")

The implication is false only if the "hypothesis" is true and the "conslusion" is false, in any other case it is true.

p:	q:	$p \rightarrow q$:
F	F	T
F	T	T
T	F	F
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Examples:
"If Warsaw is the capital of Poland then elephant is a kind of bird"

Implication (conditional statement)

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"If Warsaw is the capital of Poland then elephant is a kind of bird" (false)

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"If $2 > 3$ then $1 < 2$"

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T	F	F
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Examples:
"If Warsaw is the capital of Poland then elephant is a kind of bird" (false)
"If $2 > 3$ then $1 < 2$" (true!)

Biconditional

$p \leftrightarrow q$: "p if and only if q", "p **iff** q"

The biconditional is true only if both operands have the same truth value.

p:	q:	$p \leftrightarrow q$:
F	F	T
F	T	F
T	F	F
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Examples:
"$2 > 3$ if and only if $1 < 2$"

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"Warsaw is the smallest city in Poland iff elephant is a bird"

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Examples:
"$2 > 3$ if and only if $1 < 2$" (false)
"Warsaw is the smallest city in Poland iff elephant is a bird" (true)

Transformations of implication

Given the implication "$p \to q$, we call the following forms as follows:

- **contraposition**: "$\neg q \to \neg p$" (it is logically equivalent to the initial implication)
- *converse* of the implication: "$q \to p$"
- *inverse* of the implication: "$\neg p \to \neg q$"

Because of the contraposition, the hypothesis is also called a **sufficient condition for** the consequence, and the consequence is called a **necessary condition for** the hypothesis ("sine qua non" condition).

Conditional statement in natural language

The conditional statement "$p \to q$" has multiple equivalent forms in natural language, i.e. it can be even "hidden" in some sentences that do not look as implication at a first look.

All the following examples of forms are logically equivalent to the implication "$p \to q$":

- "p implies q", "if p, then q", "if p, q", "q if p"
- "q when p", "q whenever p"
- "p is a sufficient condition for q"
- "q is a necessary condition for p"
- "q unless $\neg p$", etc.

Precedence of logical operators in compound propositions

The usage of multiple propositional variables and operators makes it possible to create arbitrarily compound propositions.

The operators have the following priorities (precedence):
$\neg$ (the highest priority), $\vee$ and $\wedge$ (middle priority), $\rightarrow$ and $\leftrightarrow$ (the lowest priority)

We can use parentheses in case of arbitrarity or to force other precedence.

E.g.: $p \vee \neg q \rightarrow \neg r \wedge s$
Is ok even without parentheses and is equivalent to:
$(p \vee (\neg q)) \rightarrow ((\neg r) \wedge s)$

But: "$p \vee q \wedge r$" is not precise since it can be interpreted as:
"$(p \vee q) \wedge r$" or "$p \vee (q \wedge r)$", etc.[1]

[1] Some conventions make precedence of $\wedge$ before $\vee$ and $\rightarrow$ before $\leftrightarrow$, but for safety it is better to use parentheses, anyway

Tautology

A compound proposition that is always true, no matter what are the truth values of its constituent propositions is called a **Tautology**[2]

Tautologies can be used to present some **general laws** of propositional calculus, e.g.:

$(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$ is a tautology (it represents the fact that any implication is logically equivalent to its contraposition), etc.

Whether a compound proposition is a tautology can be checked with the **truth table method** i.e. checking all possibilities of truth values of the constituent propositions.

[2]otherwise it is called a *contradiction* (always false) or *contingency* (the remaining case)

Example: tautology testing with a truth table

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$(p \to q) \leftrightarrow (\neg q \to \neg p)$:

p	q	l:$p \to q$	r: $\neg q \to \neg p$	$l \leftrightarrow r$
F	F	T	T	**T**
F	T	T	T	**T**
T	F	F	F	**T**
T	T	T	T	**T**

Example: tautology testing with a truth table

$(p \to q) \leftrightarrow (\neg q \to \neg p)$:

p	q	l: $p \to q$	r: $\neg q \to \neg p$	$l \leftrightarrow r$
F	F	T	T	**T**
F	T	T	T	**T**
T	F	F	F	**T**
T	T	T	T	**T**

Note: if there are 3 variables there are 8 rows (cases) to test all combinations, for 4 variables there are 16 rows, etc. (always 2^n, where n is the number of different propositional variables)

Logical Equivalence

Two compound propositions p and q are called **logically equivalent** if $p \leftrightarrow q$ is a tautology.

We use the denotation $p \equiv q$ (or "$p \Leftrightarrow q$") for such case.

Note: "$\equiv$" (or "$\Leftrightarrow$") is not an operator of propositional calculus[3]. Rather, "$p \Leftrightarrow q$" means that "compound proposition q **is logically equivalent** to p".

Logical equivalences are used to represent some general laws of propositional calculus.

Examples:
$(p \rightarrow q) \equiv (\neg q \rightarrow \neg p)$ (contraposition)
$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$ (distributive law)

Truth tables can be used to prove logical equivalences.

[3] It is a part of a *meta-language* of propositional calculus, i.e. it serves to **describe** it

Logical Equivalences (general laws of propositional calculus)

Equivalence:	Name:
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Equivalence:	Name:
$\neg\neg p \equiv p$	double negation law
$p \lor \neg p \equiv T$ $p \land \neg p \equiv F$	negation laws
$p \land T \equiv p$ $p \lor F \equiv p$	identity laws

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$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$	negation laws
$p \wedge T \equiv p$ $p \vee F \equiv p$	identity laws
$p \vee T \equiv T$ $p \wedge F \equiv F$	domination laws

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$p \vee p \equiv p$ $p \wedge p \equiv p$	idempotent laws

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$p \vee T \equiv T$ $p \wedge F \equiv F$	domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	idempotent laws
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	commutative laws

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$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	associative laws

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$p \lor T \equiv T$ $p \land F \equiv F$	domination laws
$p \lor p \equiv p$ $p \land p \equiv p$	idempotent laws
$p \lor q \equiv q \lor p$ $p \land q \equiv q \land p$	commutative laws
$(p \lor q) \lor r \equiv p \lor (q \lor r)$ $(p \land q) \land r \equiv p \land (q \land r)$	associative laws
$(p \lor q) \land r \equiv (p \land r) \lor (q \land r)$ $(p \land q) \lor r \equiv (p \lor r) \land (q \lor r)$	distributive laws

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$p \vee p \equiv p$ $p \wedge p \equiv p$	idempotent laws
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	associative laws
$(p \vee q) \wedge r \equiv (p \wedge r) \vee (q \wedge r)$ $(p \wedge q) \vee r \equiv (p \vee r) \wedge (q \vee r)$	distributive laws
$(\neg(p \wedge q) \equiv (\neg p \vee \neg q)$ $(\neg(p \vee q) \equiv (\neg p \wedge \neg q)$	De Morgan laws

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$(\neg(p \wedge q) \equiv (\neg p \vee \neg q)$ $(\neg(p \vee q) \equiv (\neg p \wedge \neg q)$	De Morgan laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (q \vee p) \equiv p$	absorption laws

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$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	associative laws
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Some logical equivalences involving implications and biconditionals

$$p \rightarrow q \equiv \neg p \vee q$$

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$p \rightarrow q \equiv \neg p \vee q$

$p \rightarrow q \equiv \neg q \rightarrow \neg p$ (contraposition)

Some logical equivalences involving implications and biconditionals

$p \to q \equiv \neg p \lor q$

$p \to q \equiv \neg q \to \neg p$ (contraposition)

$\neg(p \to q) \equiv p \land \neg q$ (negation of implication)

Some logical equivalences involving implications and biconditionals

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$\neg(p \to q) \equiv p \wedge \neg q$ (negation of implication)

$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

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$(p \to q) \land (p \to r) \equiv p \to (q \land r)$

$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$

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$(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$ (negation of biconditional)

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$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$ (negation of biconditional)

Summary

- Proposition
- Logic operators
- Tautology
- Logical Equivalence
- Laws of propositional calculus

Example tasks/questions/problems

- give the definition of proposition and truth value of a proposition
- give 2 examples of statements that are propositions and 2 that are not
- give the name, denotation, interpretation and truth table for each of the discussed operators
- for each operator give an example of a natural language sentence that illustrates it
- what are the names of the operands of the implication operator?
- list at least 5 different ways of expressing "$p \to q$"
- what is tautology? what is logical equivalence?
- learn by heart and list the discussed logical equivalences
- prove the selected 2 tautologies and logical equivalences using truth tables
- take 3 compound natural sentences and translate them to mathematical form by defining its constituent components as propositional variables and using operators
- for a given compound proposition give an example of a natural language sentence that represents it

Thank you for your attention.